Macroeconomics Comprehensive Exam
August 2007

ID Number________

You will receive four bluebooks, numbered 1 through 4. Put your ID Number (not your name) on the line above and on the cover of each bluebook.

You must answer all questions below. Each question is worth 25%. Use a separate bluebook for each question.

You have four hours to complete your exam.

Question 1 (Answer in Bluebook #1).

Write down and illustrate graphically the steady state of the economy in the standard Solow model of economic growth. In answering, consider the following:

a. The effect of an increase in the saving rate on per-capita output and economic growth.

b. The manner in which a natural resource constraint, when added to the model, can exert a "growth drag." How important is this drag likely to be?

Question 2 (Answer in Bluebook #2).

Consider the Ramsey-Kass-Koopmans model with infinitely-lived households.

a. Write down the optimization problem of the household. Do not forget to include all the relevant conditions.

b. Write down the Hamiltonian and derive the Euler equation.

c. Discuss the difference between this model and the standard Solow model with respect to the attainability of the Golden-Rule rate of saving.

Question 3 (Answer in Bluebook #3).

The demand for money is assumed to be given by $M_t^d / P_t = E_t \{P_{t+1}/P_t\}^{-\eta}$. The supply of money is exogenous and given by M_t. Note that variables indexed by *t* are measured at the end of the period *t*.

a. Write down the equation (in logs) describing the equilibrium in the money market.

b. From now on, assume perfect foresight. Rewrite the equation in (a) under this assumption.

c. Derive the expression for the period-*t* price level as a function of the current and future money supply m_s, $s = 1,...,\infty$. To prevent speculative bubbles, assume that $\lim_{T\to\infty}(\eta/(1+\eta))^T p_{t+T} = 0$.

Using the result that $\sum_{s=t}^{\infty}\frac{1}{1+\eta}\left(\frac{\eta}{1+\eta}\right)^{s-t} = 1$, do the following:

d. Show that when the money supply is constant at m*, the price level is also constant. Find the equilibrium price level in this case.

e. There is an unanticipated announcement at $t = 0$ of a money supply increase at $t = T$, so that

$$m_t = \begin{cases} m^0, \text{ if } t < T \\ m^1, \text{ if } t \geqslant T \end{cases}$$

Derive the path of the price level starting at $t = 0$. Plot the trajectory of the price level on a graph. Show what happens at times $t = 0$ and $t = T$.

<u>Question 4 (Answer in Bluebook #4).</u>

An economy is populated by a representative consumer with lifetime utility given by $\sum_{t=0}^{\infty}\beta^t(k_t'Rk_t + l_t'Ql_t)$, where k_t is capital stock and an $n\times 1$ vector, l_t is leisure and $m\times 1$, R is $n\times n$ and symmetric, Q is $m\times m$ and symmetric, and $0<\beta<1$ is the discount factor. The evolution of the capital stock is given by $k_{t+1} = Ak_t + Bl_t$, where A is $n\times n$, and B is $n\times m$. Matrices R, Q, A, and B are parameters.

The consumer maximizes his lifetime utility subject to the motion equation of the capital stock. This question asks you to solve this optimization problem using dynamic programming. Note that the value function in this case will be quadratic, in the form of $V(k) = k'Pk$, where P is an unknown symmetric $n\times n$ matrix.

a. What is the state variable in this problem? What is the control variable?
b. Write down the Bellman equation for this problem.
c. Take the first-order condition and derive the policy function.
d. Plug the policy function into the Bellman equation and derive the (implicit) equation for P (the Riccati equation).
e. Suggest numerical methods that can be used to solve the Riccati equation.