Department of Economics, Suffolk University
Ph.D. Comprehensive Examination, Microeconomics

August 13, 2008

Please answer all the questions. The exam is designed to last for four hours. Explanations should be clear and concise. Show all of your calculations. In answering the questions, please use the blue books distributed during the testing period.

1. (50 minutes)

A price-taking consumer has income M and utility function $U(x_1,x_2) = x_1 \cdot x_2$, and faces a price vector $p = (p_1,p_2)$ for goods 1 and 2.

a. Find explicit solutions for the consumer's
   i. Marshallian demand functions;
   ii. Indirect utility function;
   iii. Hicksian demand functions; and
   iv. Expenditure function.
   v. Comment briefly on the plausibility of these functions.
   vi. Use the equations computed here to show that the Slutsky equation holds.

b. Suppose M=20 and $p_1 = p_2 = 1$. Compute the value of the consumer's utility.

c. Now suppose that p_2 rises to 2.25 while p_1 remains at 1. Thus the cost of living has increased.
   i. One measure of the cost of living is a base-weighted price index defined as the ratio of the expenditure required to purchase the original bundle under the new prices to the expenditure required to purchase the original bundle under the original prices. Compute this measure.
   ii. An alternative is to compute a true cost-of-living index by asking how much expenditure would be necessary in order to maintain utility at its original level, given the increase in the price of p_2. The cost-of-living index is then defined as the ratio of this required expenditure relative to the original expenditure level. Compute this measure.
   iii. Explain briefly why the true cost-of-living index (from ii.) rose by less than the base-weighted index (from i.).

2. (40 minutes)

A firm has a production function of the form

$$Q(L) = L^{\alpha}$$

where $\alpha \in (0,1)$, and L is the amount of labor employed. The firm operates in a competitive product market, where it sells its output at price $p = 1$. The firm's only input is labor, which it purchases at a wage $w(L)$, which is allowed to depend on the amount of labor employed.

a. Suppose that the wage is constant, so $w(L) = v$. Find the optimal employment L^* for the firm.

b. Now suppose that the wage rate is given by $w(L) = vL^{1-\beta}$, where $\beta \in (0,1)$ and $v > 0$, which means that the firm is a dominant employer in the labor market. Find the optimal employment $L^*(\beta,v,\alpha)$ as a function of β, w and α.

c. In which situation – i.e. a. or b. – is the firm's demand for labor more responsive to a given proportional change in the wage parameter, v? Explain.

3. (40 minutes)

A consumer has preferences over two goods, x and y. His income consists of an endowment of 8 units of x and 2 units of y. For each of the utility functions below, (i) draw a graph with some sample indifference curves, and (ii) find the consumer's offer curve (given his endowment), and draw it on a second graph. [Reminder: a consumer's offer curve describes the locus of tangencies between the indifference curves and the budget line, as relative prices vary.] For part (i), you just need to draw curves with the correct shapes; for part (ii) you should label your graph clearly, including the endowment point, equations for the curve(s), and points of intersection (if the curve has multiple segments).

a. $u(x,y) = xy$

b. $u(x,y) = x + y + \min\{x,y\}$.

4. (45 minutes)

Consider a set of individuals, $i \in \{1,2,....,N\}$, and two goods, salt and money. Assume that each individual has quasilinear preferences represented by the utility function $U(m,s) = m + \log(s)$, where s is the quantity of salt and m is the amount of money consumed by the individual (there are no savings in the model). Assume that each i has an initial endowment given by $\omega_i = (m_i, s_i) >> 0$.

a. Calculate i's net demand, and the aggregate net demand, for salt.

b. If it exists, calculate the Walrasian equilibrium for this economy. If it does not exist, clearly explain why.

5. (65 minutes)

Two roommates, A and B, have just moved into a two-bedroom apartment. Each of them cares only about the amount of money he has and the number of posters on the walls of the living room. More specifically, each $i \in \{A, B\}$ has the utility function $U_i(m_i, Q) = m_i + 2\alpha\sqrt{Q}$ where m_i denotes the amount of money (in dollars) that i has, Q denotes the **total** number of posters on the living room wall, and $\alpha > 0$. Currently each roommate (only these two are living in this apartment) has m dollars and there are no posters. The kinds of posters they like are sold at a unit price of one dollar. While answering the questions below, assume that one can buy a fraction of a poster and that the living room is huge, i.e., there is neither an integer constraint nor an upper bound on the number of posters.

a. Calculate the total quantity of posters, Q^*, that would maximize the sum of the roommates' utilities.

b. Suppose that each person decides independently and simultaneously on how many posters, q_i, to buy. Model this situation as a simultaneous move game and calculate its equilibria. How many equilibria does this game have? How many posters will the roommates end up with? Compare your answer with the optimal number of posters from (a). Discuss the intuition.

c. Suppose that the roommates will play the symmetric equilibrium in (b), i.e., $q_A = q_B$. Knowing this, the roommates' landlord, C, offers them the following option before any purchase is made: C will buy Q^* posters and put them on the walls, but each of the roommates will pay *R* dollars to C. What is the highest *R* that will be accepted by both roommates?

d. Ignore parts (b) and (c), and suppose that the roommates agree that on the first day, A will buy as many posters as he wants and then on the second day, after observing q_A, B will buy as many as he wants (they both agree that nobody will buy any posters before or after the day assigned to him). How many posters will each buy?

e. Now assume that A does not know whether or not B likes the posters. More specifically, suppose that A thinks that B's utility function is $U_B(m_B, Q) = m_B + 2\alpha\sqrt{Q}$ with probability ½ and that it is $U_B(m_B, Q) = m_B$ with probability ½. In addition, assume the game is played as described in part d. How many posters will A buy on the first day?